Analysis of Fully Stalled Compressor

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The present study examines the stalled operation of 27 single-stage and multistage compressor configurations. An approximation of paddle-wheel operation is used to suggest a zero-throughflow model that, when combined with an empirical correction factor, would predict the pressure coefficient of compressors at shutoff. The derivation indicates that three compressor-rotor design parameters (hub/tip ratio, aspect ratio, and setting angle) influence the values of the pressure coefficient when compressor flow is close to zero. The analytical modeling so far derived indicates the general trend of the variation of the pressure coefficient with the compressor three design parameters, but comparison with experimental data shows considerable scatter, leaving room for improvement. It is hoped that this study will encourage others to pursue more in-depth examination of stalled compressor operation.

Introduction

UNTIL a few years ago, knowledge of compressor performance in highly throttled regimes was very limited. Recently, however, the rotating stall and, more particularly, the pressure rise in a compressor stage and in multistage compressors operating with closed throttle have been studied and reported extensively in Ref. 1. The authors conclude that total-to-static pressure rise per stage of a closed-throttle compressor is largely independent of the compressor design. In other words, at closed throttle all axial flow compressors tend to achieve the same average pressure rise per stage. This average pressure coefficient, based on blade velocity at the blade pitch line, was 0.11 for 13 different low-speed compressors having hub/tip ratios of 0.8. The derivations from this mean were not small, ranging from +27 to −36% of this value. The published design characteristics (design flow coefficient, camber, blade-setting angle, and degree of reaction) of the 13 compressors were insufficient to permit an assessment of the significance of these deviations; therefore, only statistical trends could be established.

Day et al.[1] concluded that an important contribution to improved compressor design would be clarification of the fluid dynamics associated with the closed-throttle condition and derivation of a model to explain the nearly constant values of the stall pressure rise coefficient per stage for axial-flow compressors.

Here we examine the stalled operation of 27 single-stage and multistage compressor configurations. Two of the configurations operate with high flow Mach numbers, typical of advanced stages. An approximation of paddle-wheel operation is used to suggest a zero-throughflow model that, when combined with an empirical correction factor, would predict the pressure coefficient of compressors at shutoff. The analytical results are compared with the closed-throttle pressure coefficients for the 27 compressors.

Experimental Data

Design data and available information on the performance of 27 single-stage and multistage compressors[2–8] tested in deep stall and with very low-flow coefficients are summarized in Table 1.

Some of the experimental performance data of low-speed and high-speed compressors are limited to flows somewhat greater than zero. Consequently, those performance characteristics had to be extrapolated down to flow coefficients of zero.

The design and performance data for all 27 compressor builds will be examined before attempting to model the operation of compressor stages at complete shutoff. It must be stressed at this time that the experimental performance data were byproducts of compressor stability studies.

Several design parameters for these compressors have been plotted as a function of the pressure rise cofficient in Fig. 1. The results reveal the absence of any well-behaved dependence of the closed-throttle pressure coefficient on the design parameters. In other words, regular variations in the data are not apparent, and no single parameter shown accounts consistently for the change in the pressure coefficient at zero-capacity condition.

Development of a Flow Model

Little is understood about the operation of compressors in deep stall, but it is generally accepted that at the closed-throttle condition, and even for very low-flow coefficients, one stall region covers the entire blade row of a compressor stage. Because of this, stall zones extend from hub to tip of the blades, and blockage of the throughflow area is nearly complete.

It will therefore be assured that the fluid trapped within the stalled zone will be impacted by the moving compressor blades. In this situation, the compressor behavior will be analogous to the churning action of the revolving plates of a paddle wheel. The idea of a paddle wheel was also advanced in Ref. 9. A simple two-dimensional formulation of energy transfer in such a type of flow may be based, most conveniently, on the principle of the change of momentum. In other words, the axial component of force exerted by blades on the fluid will be equal to the total change of fluid momentum in that direction.

Consider a "jet" of air impinging on a blade as shown in Fig. 2. The component of the inlet momentum perpendicular to the blade equals $\rho \cdot \text{area} \cdot u^2 \cdot \sin^2\theta$, where area indicates the jet cross section, u is the jet speed that in this case is the blade tangential pitch line or average velocity, and θ is the blade-setting angle (measured to the tangential direction).

The component of the leaving momentum perpendicular to the blade is identically zero because the leaving flow is

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Table 1 Design and performance data on 27 compressor builds

		Compressor							
		Design					Performance		
Designation number	Reference	Hub/tip ratio, C	Solidity, σ	Aspect ratio, AR	Stagger angle, θ	Number of stages, η	Mean blade speed, u_m	Design flow parameter, φD	Experimental pressure coefficient, Ψ_0
1	2	0.6	0.995	2.7	50.0	1	118	0.45	0.226
2	2	.6	.862	2.7	45.5	1	118	.45	.214
3	2	.6	.862	2.7	45.5	3	118	.45	.195
4	3	.7	1.24	2.0	63.7		279	.55	.158
5	4	.88	1.373	.8	51.4		236	.6	.128
6	4	.88	1.373	.8	46.4		236	.6	.145
7	5	0.8	1.06	2	40	1	165	0.35	0.137
8			1.06		40	3		.35	.135
9			1.27		55	1		.55	.100
10						2			.105
11						3			.103
12						4			.104
13						1		.71	.075
14						2		.71	.125
15						3		.71	.117
16			1.13		70	1		1.0	.066
17						2			.100
18						3			.106
19						4			.110
20	6	0.7	0.97	1.83	48	4	144	(a)	0.167
21		.85	1.11	1.25	47.8		171		.131
22		.7	1.07	2.0	57		144		.156
23			.606	4.6	57				.125
24			1.08	4.6	50				.111
25			1.47	2.8	57				.111
26	7	0.53	1.77	3.1	39	1	1064	0.46	0.145
27	8	.63	1.35	1.0	41.2	3	937	.5	.10

[a]Not available

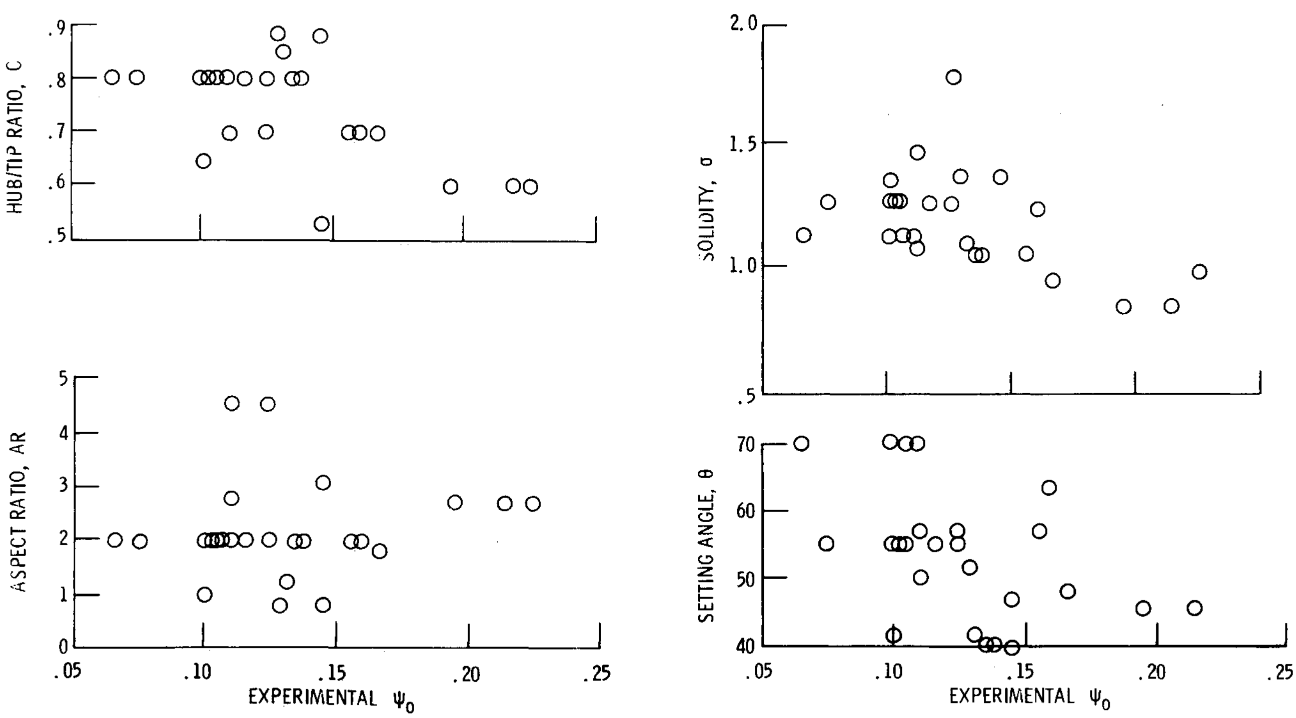

Fig. 1 Compressor design parameters vs experimental pressure coefficient at zero flow.

tangent to the blade. As force equals inlet momentum minus leaving momentum, the force normal to the blade is $F = \rho \cdot \text{area} \cdot u^2 \cdot \sin^2\theta$ and its axial component is

$$F_{ax} = \rho \cdot \text{area} \cdot u^2 \cdot \sin^2\theta \cos\theta$$

Before going any further, let us assume that the area under consideration equals the blade area hc, height times chord, and the velocity u is an average over the blade. Then

$$u^2 = \tfrac{1}{2}(u_t^2 + u_h^2) = \tfrac{1}{2}u_t^2(1 + C^2)$$

where $C = r_h/r_t$. Substituting yields

$$F_{ax} = \tfrac{1}{2}\rho(hc)u_t^2(1 + C^2)\sin^2\theta \cos\theta$$

It is not known how uniformly this force is distributed over the blades and the compressor cross-sectional annular area $\pi(r_t^2 - r_h^2)$. It may be assumed for the time being that F_{ax} acts on the entire area and that it reflects a pressure difference across the compressor blade row.

$$\Delta p = \frac{\text{force}}{\text{area}} = \frac{F_{ax}}{\pi(r_t^2 - r_h^2)}$$

or

$$\Delta p = \tfrac{1}{2}\frac{\rho(hc)}{\pi(r_t^2 - r_h^2)}u_t^2(1 + C^2)\sin^2\theta \; \phi\vartheta\lambda\theta$$

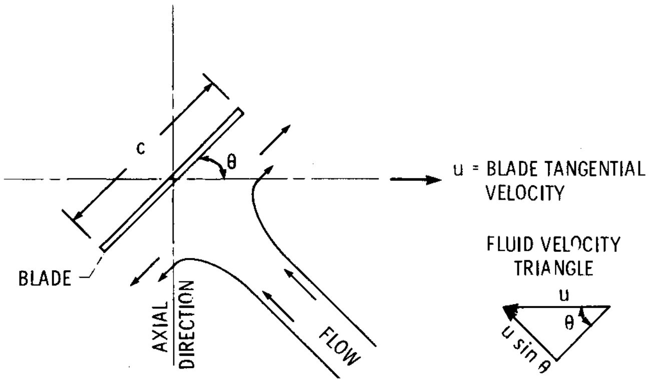

Fig. 2 Schematic of rotor blade operation in shutoff condition.

Now

$$\frac{hc}{(r_t^2 - r_h^2)} = \frac{(1 - C)^2}{AR(1 - C^2)}$$

so that

$$\Delta p = \left[\frac{k}{(2\pi AR)}\right](\rho u_t^2)(1 + C^2)\frac{(1 - C)^2}{(1 - C^2)}\sin^2\theta \cos\theta$$

and since

$$u_m = \tfrac{1}{2}(1 + C)u_t$$

$$\psi_0 = \frac{\Delta p}{(\rho u_m^2)} = \frac{2k}{[\pi AR]}(1 + C^2)\frac{(1 - C)^2}{(1 + C)^2(1 - C^2)}\sin^2\theta \cos\theta$$

where k is a constant intended to compensate for the distribution of forces on the two unknown effective areas taken into consideration. Its value ($k = 15.75$) was determined by matching only one data point of the 27 known experimental values of ψ_0. The data point was determined on the basis of best agreement for all 27 compressors. The calculated closed-throttle pressure coefficients are shown in Tables 2 and 3 using this value of k.

Modeling of the two transonic compressors required that on top of the derived momentum exchange process a correction be introduced to account for the existence of shocks in the blade passages. Using the obtained relation for ψ_0, the uncorrected values for the pressure coefficient for the two compressors are given in the third column of Table 3. These values are much higher than the experimental data. We may thus postulate that shocks prevent the rotating blades from producing the calculated pressure rise. In view of this, we proposed to use a correction factor $\sqrt{M^2 - 1}$, where the Mach number M is

Table 3 Modeling of the pressure coefficient at shutoff Ψ_0 for two transonic compressors

Compressor	Experimental Ψ_0	Calculated Ψ_0: Without correction	Calculated Ψ_0: Including $\sqrt{M^2-1}$
26	0.145	0.164	0.128
27	.10	.367	.112

Table 2 Comparison of calculated and experimental values of the pressure coefficient at zero flow for 25 low-speed compressors

Pressure coefficient at shutoff, Ψ_0					
Measured	Calculated		Measured	Calculated	
Experimental pressure coefficient, Ψ_0	Pressure coefficient, Ψ_0	Discrepancy, Δ, %	Experimental pressure coefficient, Ψ_0	Pressure coefficient, Ψ_0	Discrepancy, Δ, %
0.226	0.186	21.5	0.125	0.108	15.3
.214	.176	22.0	.117	.108	7.9
.195	.176	10.9	.066	.085	28.8
.158	.164	2.7	.100	↓	17.6
.128	.153	19.5	.106	↓	24.7
.145	.145	0	.110	↓	29.4
0.137	0.089	54.3	0.167	0.184	10.1
.135	.089	51.3	.131	.108	21.7
.100	.108	8.4	.156	.175	12.2
.105	↓	3.2	.125	.075	63.8
.103	↓	5.2	.111	.075	48
.104	↓	4.2	.111	.125	12.7
.075	↓	44.5			

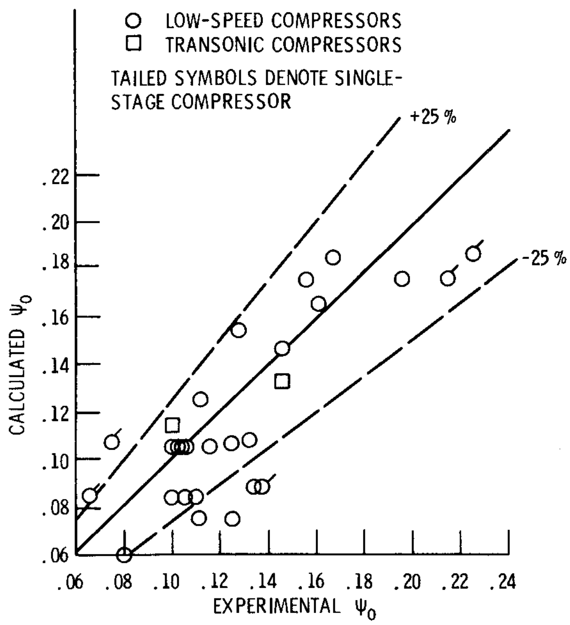

Fig. 3 Verification of validity of established flow model.

based on blade tip velocity. As indicated in Ref. 10, pressure differences on an area subject to supersonic flows depend on the parameter $\sqrt{M^2-1}$. The fourth column of Table 3 gives the new corrected values of ψ_0 for the transonic compressors.

The calculated values of the closed-throttle pressure coefficient are plotted against the experimental values in Fig. 3. The vertical distance between a point and the 45 deg line represents the deviation between calculated and experimental values. As seen from the dashed lines, which represent ±25% deviation, most of the calculated values are within 25% of the experimental values. Although there is a significant amount of scatter in the correlation, the model does indicate the trend of increasing calculated values with increasing experimental values. Thus, the model reflects the influence of compressor-rotor hub/tip ratio, aspect ratio, and setting angle on the closed-throttle pressure coefficient.

Caution must be exercised in using this correlation, especially for transonic compressors, because the Mach number correction was based on only two available data points. It must be recognized that this correlation is a preliminary one and will require future adjustments. However, it is believed that this correlation represents an improvement over the previous assumption that the closed-throttle pressure coefficient was a constant independent of compressor geometry.

The established relationship was based on data for single stages and stage averages for multistage compressors. As stated before, the calculated ψ_0 pertain to the normalized pressure rise per stage. In multistage compressors of n stages, the overall compressor ψ_0 will be n times the stage (average stage) value. Values of the stage pressure coefficient are simply additive because their evaluation was based on axial forces that are additive. In some situations when multistage compressors are considered, the first and last stage may have quite different ψ_0 than the average because of the effect of inlet and outlet plenum volumes.

The analytical model thus far obtained does not account for compressor speed because the adopted incompressible fluid flow model does not yield the dependence of the pressure coefficient on rotational speed. The experimental data do indicate, however, that such a dependence exists. With the low-speed compressors, the compressibility effects are small; with the transonic compressors, they are quite pronounced.

Concluding Remarks

With a momentum exchange model, an analytical expression for a compressor stage pressure coefficient has been obtained for the closed-throttle condition. In this equation, an empirical constant was determined using design and performance data for one low-speed compressor. The derived expression for the pressure coefficient was then evaluated using data from several low-speed and two transonic compressors.

The derived correlation reveals that for low-speed single-stage and multistage compressors, as well as for transonic compressors, the pressure rise coefficient per stage at closed-throttle condition is not a constant, and its value is influenced by the hub/tip ratio, blade aspect ratio, and the setting angle of the rotor blades. For the transonic compressors, compressibility effects must also be considered.

The analytical modeling so far derived needs further development. The adopted model indicates the general trend of the variation of the pressure coefficient with compressor design parameters. Most of the calculated values for the closed-throttle pressure coefficient are within 25% of the experimental data. Although there is scatter in the correlation, it does represent a marked improvement over the previous assumption that the closed-throttle pressure coefficient was a constant independent of compressor geometry.

Possible improvements to the correlation may include the effect of stator geometry and perhaps that of the inlet guide vanes. Most useful in this respect would be tests run with the explicit intention of evaluating the effect of compressor geometry on closed-throttle compressor operation. Thus far, no such tests were ever run.

References

[1]Day, I. J., Greitzer, E. M., and Cumpsty, N. A., "Prediction of Compressor Performance in Rotating Stall," *Journal of Engineering for Power,* Vol. 100, Jan. 1978, pp. 1-14.

[2]Iura, T. and Rannie, W. D., "Observations of Propagating Stall in Axial-Flow Compressors," Rept. 4, Mech. Eng. Lab., California Institute of Technology, Pasadena, April 1953.

[3]Greitzer, E. M., "Surge and Rotating Stall in Axial Flow Compressors: Part II, Experimental Results and Comparison With Theory," *Journal of Engineering for Power,* Vol. 98, April 1976, pp. 199-217.

[4]Gamache, R. N., "Axial Compressor Reversed Flow Performance," Ph.D. Dissertation, Massachusetts Institute of Technology, Cambridge, 1985.

[5]Day, I. J., "Axial Compressor Stall," Ph.D. Dissertation, Christ's College, Cambridge Univ., 1976.

[6]Wisler, D. C., prive communication, General Electric Co., Evendale, OH.

[7]Urasek, D. C., Steinke, R. J., and Cunnan, W. S., "Stalled and Stall-Free Performance of Axial-Flow Compressor Stage with Three Inlet-Guide-Vane and Stator-Blade Settings," NASA TN D-8457, 1977.

[8]Mason, J. R., "Post-Stall Compressor Model, P7WA/GPD/FR-16286, Pratt and Whitney Aircraft, West Palm Beach, FL, Jan. 1983 (NASA CR-168048).

[9]Day, I. J. and Cumpsty, N. A., "The Measurement and Interpretation of Flow Within Rotating Stall Cells in Axial Compressors," *Journal of Mechanical Engineering Science,* Vol. 20, April 1978, pp. 101-115.

[10]Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow,* Roland Press Co., NY, 1953.